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The Surface Area of Some Selected Simple Shapes

For the practicing surface finisher, it is extremely important to know the surface area of the items to be plated. For example, the surface finisher may know that he can plate, say, 500 pounds per electroplating barrel, but without knowing the surface area, the surface finisher doesn't know how long the parts will take to plate to the desired plating thickness. And, without knowing the surface area, the surface finisher will not know the chemical, metal and material costs involved in finishing the items. And without knowing the cost, pricing may or may not result in profit for the surface finisher.

Most surface finishers base their pricing primarily on throughput - in other words, attempting to generate a certain amount of money per barrel or per rack or per hour. But in today's economic environment of permanently inflated material costs, a deeper knowledge of costs based on materials and surface area is required. And that understanding begins with the study of surface area.

And before we get too involved in this analysis, what we will be analyzing and discussing is the surface area per unit weight and not merely the surface area *per se,* because much of the surface finishing costing and pricing is done on a weight basis.

To determine the surface area per unit weight, we follow a clearly defined calculation protocol. While the procedure which follows is straightforward and simple, actual implementation can be complex, demanding and tedious.

First Step	Calculate the exposed surface, adding all elements.
Second Step	Calculate the volume of the piece, adding all elements.
Third Step	Multiply the volume of the piece by the density of the material of construction.
Fourth Step	Divide the surface area by the weight.
Fifth Step	Correct or recalculate the result of the fourth step to the desired system of units.

Circular flat blank

Let us consider, first, an extremely simple shape, albeit one that is seldom plated - a circular flat blank. The most common plated part of this nature is the common penny, which is a stamped zinc blank plated with copper.

In the discussion which follows, we will ultimately calculate the surface area in square feet per 100 pounds (ft^2/cwt). This normally results in a number between 1 and 500 that is readily understood. Typically platers talk in terms of amps per square foot and painters talk of paint coverage as mils per square foot per gallon. For those who prefer to work in the SI System (metric), square feet per hundred pounds can be converted to square meters per kilogram by multiplying ft^2/cwt by 0.002048 (or alternatively, dividing by 488.26).

We can use the protocol above to calculate the surface area per unit weight for this shape, as the following calculation shows. We will consider a circular flat blank with a diameter d and a thickness t.

First Step: (Calculating the exposed surface area)

$$A = 2\pi\left(\frac{d}{2}\right)^2 + \pi dt \tag{1}$$

which is, mathematically, the area of the circle (πr^2) doubled (because we are considering both the top and the bottom), plus the edge area (the circumference πd multiplied by the thickness t).

Second Step: (Calculating the volume)

$$V = \pi\left(\frac{d}{2}\right)^2 \times t \tag{2}$$

which is the area of the top surface (πr^2) multiplied by the thickness (t).

Third Step: (Calculating the weight)

$$w = 0.2833 \times V \tag{3}$$

Note that here we are using a density of 0.2833 lb/in^3, which is typical for steel (For copper it would be 0.3223 lb/in^3). If other materials of construction are used, the appropriate density should be used.

Here are a few of the more common densities that are encountered. Data is readily available from standard reference materials such as *The Handbook of Chemistry and Physics* and *Machinery's Handbook*.

Metal	Density, g/cm^3	Density, lb/in^3
Aluminum	2.71	0.98
Yellow Brass	8.47	3.06
Phosphorus Bronze	8.78	3.17
Copper	8.92	3.22
Hastelloy	9.25	3.34
Inconel	8.03	2.90
Monel	8.36	3.02
Stainless Steel	8.03	2.90
Steel	7.84	2.83
Wrought iron	7.75	2.80
Zinc	7.14	2.67

For example, later in this article we will calculate the surface area of a plain washer at 68.64 ft^2/cwt. The same washer in aluminum would have the same surface area per washer but 2.89 times as much surface area per pound or 198.37 ft^2/cwt.

Fourth Step: (Calculating the surface area per unit weight)

The area *A* per unit weight *W* is then given by:

$$\frac{A}{W} = \frac{\pi\left(\frac{d}{2}\right)^2 + \pi dt}{0.2833 \times \pi\left(\frac{d}{2}\right)^2 \times t} \tag{4}$$

which, after changing the units from square inches per pound to square feet per hundred pounds, simplifies to:

Fifth Step: (Converting to the preferred system of units)

$$\frac{ft^2}{cwt} = \frac{4.925}{t} + \frac{9.8051}{d} \tag{5}$$

So, using a flat blank with diameter of 1.00 inches and a thickness of 0.100 inches as an example, 100 pounds of steel parts have an area of 58.83 square feet. Knowledge of the relationship between surface area and weight is an important part of mastering the economics of surface finishing.

The common washer

One of the most common part types encountered in the surface finishing business is the common washer, which is only slightly more complicated than the flat blank we just finished discussing. The common washer is used to distribute the load in a nut-bolt-washer assembly to provide a more consistent mechanical connection.

We can use the protocol above to calculate the surface area per unit weight for a common washer, as the following derivation shows. We will consider a common washer with an inside diameter d_1, an outside diameter d_2 and a thickness *t*.

First Step: (Calculating the exposed surface area)

$$A = 2\pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right] + \pi d_1 t + \pi d_2 t \tag{6}$$

which is, mathematically, the area of the outer circle minus the area of the inner circle, doubled (because we are considering both the top and the bottom), plus the area of the inner radius plus the area of the outer radius.

Second Step: (Calculating the volume)

$$V = \pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right]t \tag{7}$$

which is the area of the top surface multiplied by the thickness.

Third Step: (Calculating the weight)

Here, the formula is identical to Eq (3) for the circular disk, using a density of 0.2833 pounds per cubic inch for steel.

Fourth Step: (Calculating the surface area per unit weight)

$$\frac{A}{W} = \frac{\pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right] + \pi d_1 t + \pi d_2 t}{0.2833 \times \pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right]t} \tag{8}$$

Take careful note that this is an extremely simple part. And even for a geometrically uncomplicated part like a common washer, the mathematics is rather involved.

Fifth Step: (Converting to the preferred system of units)

After changing the units from square inches per pound to square feet per hundred pounds, the formula simplifies to:

$$\frac{ft^2}{cwt} = \frac{4.9025}{t} + 9.8051\left[\frac{d_1 + d_2}{d_2^{\,2} - d_1^{\,2}}\right] \tag{9}$$

So, using a washer with a thickness of 0.100, an inside diameter of 0.5 inches, and an outside diameter of 1.00 inch as an example (admittedly, not a standard size, but one chosen to illustrate the calculation), 100 pounds of steel parts have an area of 68.64 square feet.

The attentive reader may wish to compare this with the formula for a circular flat blank previously discussed. If we set the I.D. in the washer formula at 0.00 we arrive at the same result as we do for a circular flat blank. This is as it should and must be.

Note that what we have done here is essentially taken 5 (actually 4.9025), divided it by the thickness (0.1), and added the edge area, which in this case is 39.7% of the area of the flat surfaces. I mention this because often there is a tendency for people who use the rule of thumb of 5 divided by the thickness (which give surface area of a stamping in square feet per hundredweight) to **underestimate** the edge area.

The lockwasher

Another quite commonly encountered part is a common lockwasher, which typically locks a bolt-nut assembly. A lockwasher is a single-turn compression spring, typically with a sharp edge that digs in to the nut that is being locked and the surface to which the nut is applied. The surface area of a conventional (*e.g.*, single-turn) lockwasher is the same as the surface area of a common washer except that the ring is broken, generating additional surface area for the same weight. (Actually, a lockwasher has a thinner inner radius than outer radius, a factor which we will ignore for now, because the effect is minimal.)

Taking the formula from Step 4, above, we add $2(d_2 - d_1)t$, which is the cross-sectional area, doubled, to the numerator:

$$\frac{A}{W} = \frac{\pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right] + \pi d_1 t + \pi d_2 t + 2(d_2 - d_1)t}{0.2833 \times \pi\left[\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2\right]t} \tag{10}$$

Simplifying a little and correcting the units to square feet per hundred pounds we have:

$$\frac{ft^2}{cwt} = \frac{4.9025}{t} + 9.8051\left[\frac{d_1 + d_2}{d_2^{\,2} - d_1^{\,2}}\right] + 6.2421\left[\frac{d_2 - d_1}{d_2^{\,2} - d_1^{\,2}}\right] \tag{11}$$

This formula could, of course, be simplified a little more, but as presented, it is clearly a derivative of the plain washer formula developed earlier. So, using a lockwasher with a thickness of 0.100, an inside diameter of 0.5 inches, and an outside diameter of 1.00 inch as an example (as with the plain washer), 100 pounds of steel parts have an area of 72.796 square feet.

As a concluding example of this technique, let us consider a rectangular flat blank - a very simple stamping - with a width w, a length l and a thickness t. Consider carefully the concept that this item will have a defined area per unit weight and that we can deform this part in many ways - bending, twisting, etc. - without changing the defined area per unit volume or the area per unit weight. And consider the extension of this concept that even significant deformations of the surface will have little effect on the surface area per unit weight. With these concepts hopefully considered by the careful reader and accepted by him or her, we will proceed with our analysis:

First Step: (Calculating the exposed surface)

$$A = 2(lw + lt + tw) \tag{12}$$

which is a mathematical representation of the six sides of this item.

Second Step: (Calculating the volume)

$$V = l \times w \times t \tag{13}$$

which is the area of the top multiplied by the thickness.

Third Step: (Calculating the weight)

Again, the formula is identical to Eq (3) for the circular disk, using a density of 0.2833 pounds per cubic inch for steel.

Fourth Step: (Calculating the surface area per unit weight)

$$\frac{A}{W} = \frac{2(lw + lt + tw)}{0.2833(l \times w \times t)} \tag{14}$$

Fifth Step: (Converting to the preferred system of units)

After changing the units from square inches per pound to square feet per hundred pounds, the formula simplifies to:

$$\frac{ft^2}{cwt} = 4.9025\left[\frac{(lw + lt + tw)}{l \times w \times t}\right] \tag{15}$$

So, for a rectangular flat blank with width of 1.00 inch and a length of 2.00 inches and a thickness of 0.100 inches as an example, 100 pounds of steel parts have an area of 56.379 square feet. The value is not much different from the circular flat blank discussed earlier (58.883 square feet per 100 pounds). This mathematical behavior is typical for items fabricated from sheet steel of the same thickness.

Conclusion

There is a straightforward protocol for calculating the surface areas of parts to be surface finished. The actual calculation can be tedious even for relatively simple parts.

In my next article on this subject

Because the calculations are complex, there are three common "Rules Of Thumb" for calculating the surface area, one of which is mentioned briefly above. In my next article, I will give these rules, show how two of them are derived, and demonstrate some ingenious uses for these Rules of Thumb. ***P&SF***